A Multi-Period Optimization Model for Cement Production, Allocation and Logistics Planning

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Abstract

This paper describes an optimization based model and Decision Support System (DSS), developed for tactical supply chain planning for a large Cement making firm in India. The DSS uses Mixed Integer Linear Programming (MILP) model labeled as Manufacturing and Logistics Planner (MLP). MLP gives a tactical plan (Monthly or Quarterly plan) for goods production and distribution which maximizes the expected supply chain-wide contribution. The multi time period model considers production costs, transportation costs, taxes such as excise duty, VAT and octroi, and numerous practical constraints, and simultaneously calculates production mix at plants, primary transport allocation and mode selection from plants to warehouses and plants to markets, inventory stocking at warehouses, and secondary transport allocation between warehouses and markets. The model is useful even for annual budget planning and financial planning. The DSS has demonstrated improvement in the logistics planning approach as compared to the rule-of-thumb based planning.

1. Introduction

This paper describes an optimization based model and Decision Support System (DSS), developed for tactical supply chain planning for a large cement manufacturing firm, ABC Ltd. in

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India. The DSS uses a Linear Programming (LP) model labeled as Manufacturing and Logistics Planner (MLP).

A common problem for a large manufacturing firm like ABC Ltd. with geographically wide distribution network is the need to find a tactical plan (Monthly or Quarterly plan) for goods distribution which will maximize the total supply chain contribution. The desired optimal plan is the one that simultaneously calculates a) production mix at plants, b) primary transport allocation and mode selection between plants and warehouses or plants to markets, c) inventory stoking at warehouses, and d) secondary transport allocation between warehouses and markets.

The MLP is a multi time period model. It gives monthly or quarterly production and distribution plan by considering production costs, transportation costs, different taxes, and practical constraints.

DSS Description

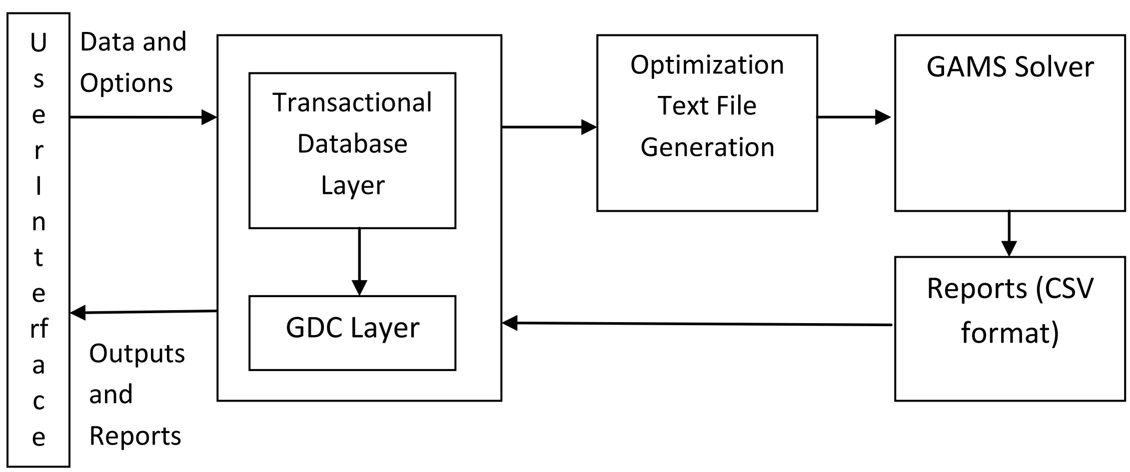

Figure 1: Steps in DSS

Figure 1 shows the broad steps of data flow in the DSS. User enters data through the front end developed in Java. Complete data is stored in the database developed in Oracle. GDC (GAMS Data Centre) layer developed in Oracle fetches relevant data from database layer as per the options selected by user in the front end, and generates text file with LP formulation and the data. A Java interface invokes GAMS execution file to run the MLP text file. After successful execution and validation of results, reports from GAMS in CSV format are dumped into the database. Important aggregate results are displayed on front end and detailed reports in excel format are stored in the path and folder specified by user.

2. Literature Review

There is extensive literature on supply chain network design and inventory optimization [1], [2]. Most of the papers discuss safety stock policy [3] and the lead time factor assuming a normal distribution demand pattern. There are also some papers which are explained about the modeling of economic scales using nonlinear [4], [5] optimization techniques. Only a handful of papers discuss a trade-off between the supply chain design and inventory optimization for tactical planning using linear programming techniques. This paper explains how the model is developed using linear programming techniques and practical constraints are applied using integer programming

3. Problem Description

The outbound supply chain network of ABC Ltd. is shown in Figure 2. It comprises of 3 echelons – Plants, Warehouses and Markets. There are 10 plants, 110 warehouses and 4300 markets. Each plant produces only 4 generic products. These products are finally segregated into different sub product as per packing which results in 24 final products (SKUs). The flow of goods/products from plant to market through warehouses is termed 'stock transfer movement' and directly from plant to market is termed 'direct movement'. Being a multi time period model, the model allows for inventory storing in warehouses.

Figure 2: Supply chain network of ABC Ltd.

For optimization, MLP takes input data like plant locations, warehouse locations, market locations, products, transport links between different nodes, transport modes, production capacities, production costs, transport costs, inventory costs and storage capacity of warehouses, and different taxes like VAT, EXCISE, CST, OCTROI etc.

The outputs from DSS are:

a) Quantity of a product to supplied from
   - A plant to a warehouse by a transport mode in each time period
   - A warehouse to a market by a transport mode in each time period
   - A plant to a market by a transport mode in each time period
   - A plant to a warehouse in period t and warehouse to market in a future period tt

b) Production plan (product mix) for each plant

c) Inventory to be stored at a warehouse in each time period

d) Total supply chain contribution, total cost and its break-up

e) Next best alternate routes for every plant-warehouse-market combination (Reduced Cost report)

4. Methodology

4.1 Model Explanation

The required sets are explained as below:

i- Plants, j- Warehouses, k- Distribution centers, q- product produced at plant, p- products demand at market label, m- different route mode of transport between factories and warehouses. n- different route mode of transport between warehouses and distribution centers, u- different route mode of transport between factories and distribution centers, set_road (m)- valid road mode, set_rail (m) - valid rail mode, t- time period, tt- alias of time, $IVT_{t,tt}$ - invalid combination of t and tt, $QD_{i,p,t}$ - Products which can be produced on dedicated equipment of production line, $VMIJ_{i,j,m}$ - Valid transport modes between i and j, $VMJK_{j,k,n}$ - Valid transport modes between j and k, $VMIK_{i,k,u}$ - Valid transport modes between i and k, Prod(q,p)- product mapping.

Parameters:

IHC_p - Inventory holding cost of product p in money unit per time period per unit of quantity, SC_j - Storage capacity of warehouse j, $ULCM_m$ - Loading and unloading cost for 1 vehicle or wagon-load of transport mode m, $ULCN_n$ - Loading and unloading cost for 1 vehicle or wagon-load of transport mode n, $ULCU_u$ - Loading and unloading cost for 1 vehicle or wagon-load of transport mode u, CTI_m - Transport cost for mode m (money unit per standard load), CTJ_n - Transport cost for mode m (money unit per standard load), CTU_u - Transport cost for mode u (money unit per standard load), $TAX1_{i,p}$ - Taxes for transporting products p from plant I, $TAX6_{j,k}$ - Warehouse and distribution center tax, $TAX7_{i,k}$ – Plant to distribution center tax, $MIN\_SHARE_{k,p,t}$ - Min share of the product, WHT_j - Warehouse tax, $CP_{i,p,t}$ - Unit cost of production of p at factory I, $PQ_{i,p,t}$- Maximum no. of units of p belonging to set QD that factory i can produce in one time- period, $IIO_{i,j,p}$ - Initial no. of units of p stored at j at the beginning of first time-period, $D_{k,p,t}$ - Demand for p in market served by k in time-period t in no. of units, PTR_SUPPLY - Total profit ignoring cost of unmet demand, $TOTAL\_COST$ - Total Cost, $TOTAL\_INVY\_COST\_P$ - Total inventory holding cost, $TOTAL\_INVY\_COST\_R$ - Total inventory holding cost of purchased items, $TOTAL\_INVY\_COST\_SUPPLY$ - Total cost excluding cost of unmet demand.

Variables:

$Y_{p,i,j,m,t}$ - No. of units of p produced at i and transported to j by mode m in time-period t, $Z_{p,j,k,n,t}$ - No. of units of p transported from j to k by mode m in time-period t, $W_{p,i,k,u,t}$ - No. of units of p produced at i and transported to k by mode u in time-period t, $IJK_{p,i,m,j,n,k,t,tt}$ - No. of units of p transported from i via j to k by primary mode m in period t and distributed from j to market k, $II_{p,j,t}$ - No. of units of p stored at j at the end of time-period t, $SHORT_{p,k,t}$ - No. of units of p whose demand is not met in market k in period t, TPR – Total profit, $not\ VATEXEM_{i,k}$ - Plant i to Market k VAT exception and ALLOC_1(i,j,m,t)-integer variable for defining the practical constraints.

Objective Function:

This model is a profit maximization model by minimizing the sum of total costs such as primary transport, secondary transport, production, inventory holding, all taxes, and shortage incurred in case the demand exceed the supply.

Total profit function is explained as follows:

$$TPR = \sum_{p,k,t} SP_{k,p,t} * \left(\left(\sum_{j,n,\in VMJK_{j,k,n}} Z_{p,j,k,n,t} \right) + \left(\sum_{i,u,\in VMIK_{i,k,u}} W_{p,i,k,u,t} \right) \right) - TOT\_COST$$

Total Cost function considered the cost parameters as follows:

TOTAL COST =

$$\sum t\ ((\sum_{i,j,m\ \in VMIJ(i,j,m)}(CTI_{m,t} + ULCM_{m,t}) * \sum_{p} Y_{p,i,j,m,t}$$

$$+ \left(\sum_{i,k,u \in VMIK\ (i,k,u)}(CTU_{u} + ULCU_{u}\right) * \sum_{p} W_{p,i,k,u,t}$$

$$+ \left(\sum_{j,k,n \in VMJK(j,k,n)}(CTJ_{n} + ULCN_{n}\right) * \sum_{p} Z_{p,j,k,n,t} + \sum_{i,q,t} CP_{i,q,t}$$

$$* \left(\sum_{p,j,m \in (prod\_mapping(q,p),VMIJ(i,j,m))} Y_{p,i,j,m,t}\right.$$

$$\left. + \sum_{p,k,u,\in (prod_{mapping(q,p)},VMIK(i,k,u))} W_{p,i,k,u,t}\right) + \sum_{p,k,t} 10E10 * SHORT_{p,k,t}$$

$$+ \sum_{i,p,t} TAX1_{i,p} * \left(\sum_{j,m \in VMIJ\ (i,j,m)} Y_{p,i,j,m,t} + \sum_{k,u,\in VMIK\ (i,k,u)} W_{p,i,k,u,t}\right)$$

$$+ \left(\left(\sum_{p,i,m,j,n,k,t,tt \in not\ IVT\ (t,tt) and\ \in VMIJ_{i,j,m}\ and\ \in VMJK_{j,k,n}} 0.103 * MRP_{p,k,t}\right)\right.$$

$$\left. * \sum_{k,p,i} IJK_{p,i,m,j,n,k,t,tt}\right) + \left(\sum_{p,i,k,u,t \in VMIK_{i,k,u}} 0.103 * MRP_{k,p,t} * W_{p,i,k,u,t}\right)$$

$$+ \left(\left(\sum_{p,i,m,j,n,k,t,tt \in (not\ IVT\ (t,tt) and\ VMIJ_{i,j,m}\ and\ VMJK_{j,k,n}\ and\ not\ VATEXEM_{i,k}} 0.125 * SP_{k,p,t}\right)\right.$$

$$\left. * \sum_{k,p,i} IJK_{p,i,m,j,n,k,t,tt}\right)$$

$$+ \left(\sum_{p,i,k,u,t \in VMIK_{i,k,u} and\ not\ VATEXEM_{i,k}} 0.125 * SP_{k,p,t} * W_{p,i,k,u,t}\right)$$

$$+ \left(\sum_{p,j,k,n,t \in VMJK_{j,k,n}} TAX6_{j,k} * Z_{p,j,k,n,t}\right) + \left(\sum_{p,i,k,u,t \in VMIK_{i,k,u}} TAX7_{i,k} * W_{p,i,k,u,t}\right)$$

$$+ WHT_{j\ and\ VMJK_{j,k,n}} * Z_{p,j,k,n,t} + TOTAL\_INVY_{COST_{P}})$$

Total profit:

$$TPR = \sum_{p,k,t} SP_{k,p,t} * \left(\left(\sum_{j,n,\in VMJK_{j,k,n}} Z_{p,j,k,n,t}\right) + \left(\sum_{i,u,\in VMIK_{i,k,u}} W_{p,i,k,u,t}\right)\right) - TOT\_COST$$

Constraints:

- Supply Constraint: Total production for each product from each plant should be less than the production capacity given.

Profit supply: $TPR\_SUPPLY = TPR + \sum_{p,k,t} 10E10 * SHORT_{p,k,t}$

Cost supply: $TOT\_COST\_SUPPLY = TOT\_COST \; - \sum_{p,k,t} 10E10 * \; SHORT_{p,k,t}$

The production system at plant label is defined as a dedicated production line. Production capacity dedicated to the plant is defined as follows:

$$capacity\_dedicated(q,i,t).. \quad \sum_{p,j,m \in (VMIJ(i,j,m) \, and \, prod_{mapping(q,p)})} Y_{p,i,j,m,t} \; + \sum_{p,k,u, \in (VMIK(i,k,u) and \, prod\_mapping(q,p)} W_{p,i,k,u,t} \;\; \leq \; PQ_{i,q,t}$$

Also there should be constraint need to be explained for the product not made in the factory as follows:

$$prod\_nt\_made\_factory(q,i,t).. \;\; \sum_{p,j,m \in (VMIJ(i,j,m) and \, prod\_mapping(q,p))} Y_{p,i,j,m,t} \; + \sum_{p,k,u, \in (VMIK(i,k,u) and \, pro\,d_{mapping(q,p)})} W_{p,i,k,u,t} \;\; \leq 10E10 \, * \, 1_{\in QD_{i,p,t}}$$

- Balance Constraint: Flow balance of a product at a warehouse. The sum of the supply from upstream in a current time period and inventory at the end of previous time period should be equal to the supply to downstream in current time period and inventory remaining at the end of current time period.

Balance constraint of plant and warehouse label for the initial inventory:

$$IIO(i,j,p) = \sum_{m,n,k,tt \, \in (VMIJ(i,j,m) and \, VMJK(j,k,n))} IJK_{p,i,m,j,n,k,"0",tt}$$

Balance constraint of plant and warehouse label for the inventory:

$$\mathrm{Balance\_IJ}_{p,i,j,m,t \, \in \, VMIJ(i,j,m)..} \;\; Y_{p,i,j,m,t} \; = \sum_{n,k,tt \, \in (not \, IVT(t,tt) and \, VMJK(j,k,n)} IJK_{p,i,m,j,n,k,t,tt}$$

Balance constraint of warehouse and destination label for the inventory:

$$\mathrm{Balance\_JK}_{p.j.k,n,tt \, \in (VMJK(j,k,n)..} \; Z_{p,j,k,n,tt} \; = \sum_{m,i,t \, \in (not \, IVT(t,tt) \, and \, VMIJ(i,j,m)} IJK_{p,i,m,j,n,k,t,tt)}$$

- Demand Constraint: Sum of supply of a product in a market through stock transfer and direct movement and the shortage must meet its demand.

$$\sum_{j,n \in VMJK_{j,k,n}} Z_{p,j,k,n,t} \; + \sum_{i,u \in VMIk_{i,k,u}} W_{p,i,k,u,t} + SHORT_{p,k,t} = \; DEMAND_{k,p,t}$$

The service level required to meet the demand is explained as by the minshare equation:

$$SHORT_{p,k,t} \leq \left(1 - MIN_{SHARE \, k,p,t}\right) * \; DEMAND_{k,p,t}$$

- Warehouse Capacity: The average inventory storage should be less than the capacity of warehouse.

Warehouse Storage Capacity First: $\sum_p \frac{\sum_i IIO_{i,j,p} + II_{p,j,1}}{(2*SJ_{j,p})} \leq SC_j$

Warehouse Storage Capacity: $\sum_p \frac{(II_{p,j,t-1\in TP_t} + II_{p,j,t\in TP_t})}{(2*SJ_{j,p})} \leq SC_j$

$$TOT\_INVY\_COST\_P = \sum_p IHC_p * \left(\sum_j (\sum_i (IIO_{i,j,p} + II_{p,j,1})\frac{1}{2}\right) + \sum_t (II_{p,j,t-1\in TP_t} + II_{p,j,t\in TP_t})\frac{1}{2})$$

- Inventory Stock Transfer: The equations linking plant to warehouse and warehouse to market movements into a single variable with plant, warehouse, market, and transport mode.

Ware house Inventory Balance First:

$$\sum_{j,m\in VMIJ\,(i,j,m)} Y_{p,i,j,m,1} + \sum_i IIO_{i,j,p} - \sum_{j,n,\in VMJK_{j,k,n}} Z_{p,j,k,n,1} - II_{p,j,1} = 0$$

Ware house Inventory Balance in time various time period:

$$\sum_{j,m\in VMIJ\,(i,j,m)} Y_{p,i,j,m,t} + II_{p,j,t-1} - \sum_{j,n,\in VMJK_{j,k,n}} Z_{p,j,k,n,t} - II_{p,j,t} = 0$$

<u>Custom (ABC specific) constraints:</u>

- Upper, lower and fixed bounds on transport mode usage on each O-D (Origin – Destination) pair explained as follows:

  $ALLOC\_RAIL\_1_{(i,j,m,t)\in(VMIJ(i,j,m) and\ set\_rail(m))} .. ALLOC\_1_{(i,j,m,t)} \geq \sum_p Y_{p,i,j,m,t} / 10E5$

  $ALLOC\_RAIL\_2_{(i,j,m,t)\in(VMIJ(i,j,m) and\ set\_rail(m))} .. ALLOC\_1_{(i,j,m,t)} \leq \sum_p Y_{p,i,j,m,t}$

  $PLANT\_WH\_RAIL_{(i,j,m,t)\in(VMIJ\,(i,j,m)\,and\,set_{rail(m)})} .. \sum_p Y_{p,i,j,m,t} = 2600 * ALLOC\_1_{(i,j,m,t)}$

- Upper, lower and fixed bounds on material transfer on each O-D pair such as

  $EQN_{1_{ROAD(t)}} .. \sum_{p,j,m\in(VMIJ("I1",j,m) and\ set\_road(m))} Y_{p,"I1",j,m,t} = 10000$

- Lower and fixed bounds on product-wise production from each plants such as

$$PLANT_{CAPACITY_t}.. \sum_{p,j,m \in (VMIJ("I1",j,m) and\ prod\_mapping(Q1,p))} Y_{p,"I1",j,m,t} + \sum_{p,k,u \in (VMIK(I1,k,u) and\ prod\_mapping(Q1,p)} W_{p,I1,k,u,t} = PQ_{"I1","Q1",t}$$

- Upper, lower and fixed bounds on supply to a cluster of markets (Sales Units) such as

$$PLANT_{DEST\_PROD(t)}.. \sum_{p,k,u \in (VMIK("I1",k,u) and\ DEST(k) and\ PROD(p)} W_{p,I1,k,u,t)} = 2000$$

4.2 Scenarios

The scenarios for which the model was run can be broadly categorized into 3 types:

Base scenario:

This scenario explains the actual allocation of demand without implementing any practical constraints. The allocation is totally based on the demand at destinations having total minimum logistic costs.

Minshare scenario:

This scenario explains the optimal allocation of demand considering the input minimum supply level at the destinations. The optimal allocation happens as per the desired supply level at the destination at the different label of the supply chain network.

Constraint scenario:

This scenario explains the optimal allocation of products at the destinations with the various levels of practical constraints. This scenario is for pure allocation problem which reflects respective business constraints.

4.3 Solution Methodology

The present MILP problem is solved using GAMS/CPLEX. CPLEX solver uses linear programming and mixed integer programming algorithm with modern algorithmic features to solve the problem. The problem size was large as per the huge data sets. For certain large dataset scenarios, in order to achieve good accuracy and reduce run time, the problem can be divided into two threads.

5. Discussion

Client uses two reports for monthly planning a) Base Run report and b) Constraint Run report. Base and Constraint reports are helpful in the beginning of the month while the Variance and

Deviation reports are generated during the mid/end of the month for analyzing the gap between actual implementation and the plan drawn at the start of the month.

The Constraint run result is used for actual planning because they represent the real life problem with actual data while the base run result gives the best result suggested by the model without any real life constraints. The comparison of constraint and base run results allows the evaluation of implementing specific constraints on the total system cost or contribution margin. The analysis of impact of real-world constraints can be used to make tactical changes like re-negotiating with transporters, changing the freight charges, changing the logistics linkages etc.

6. Conclusion

The MLP DSS aids middle management to make monthly or quarterly production and logistics plans. The decisions given by DSS are directly usable as the DSS gives the quantity to be supplied by each mode of transport in each time period between each O-D pair. This model is useful even for annual budget planning and financial planning as the time period is user defined and can be changed to one year or any other definition. The Reduced Cost report gives the next best alternate routes between an O-D pair if the optimal route is not available because of an unexpected event.

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